

# 1060803 對數 4

總	分

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## 答 案

### 一、填充題 (29 格 每格 0 分 共 0 分)

1.95 2.4 3.2 4.3 5.1或8 6.1 7.9 8.  $\frac{50}{3}$  9.  $3 < x < \frac{9}{2}$ , 且  $x \neq 4$  10.3 11.20 12.(1)3;(2)  $\frac{8}{3}$  13.4 14.  $\frac{15}{2}$  15.(1)  $\frac{10}{3}$  ;(2) -5;(3)  $\frac{9}{2}$  ;(4)4;(5)  $\frac{3}{4}$  16.  $\frac{13}{12}$  17.2 18.5

19.  $\frac{11}{3}$  20.  $a < b < c$  21.  $c < b < a$  22.  $0 < x < 3$  23.  $4 < x < 9$  24.(1)  $x > 1$  ;(2)  $\sqrt{2} < x < 4$  25.  $a < c < b$  26.  $\frac{1}{3} < x < \frac{2}{3}$  27.  $x > 5$  28.  $\frac{1}{2} < x < 3$  29.(1)  $1 < x < 3$  ;(2)

$$\frac{3}{2} < x < 6$$

## 解 析

### 一、填充題 (29 格 每格 0 分 共 0 分)

2.原式  $\Rightarrow \log_{10}[(x+1)(x-2)] = \log_{10}10$   
 $\Rightarrow (x+1)(x-2) = 10 \Rightarrow x^2 - x - 12 = 0$   
 $\Rightarrow (x-4)(x+3) = 0$   
 $\Rightarrow x = 4$  或  $-3$  (不合), 故  $x = 4$

3.原式  $\Rightarrow \log_5 \frac{7x+10}{2x-1} = \log_5 8$   
 $\Rightarrow \frac{7x+10}{2x-1} = 8 \Rightarrow x = 2$

4.  $\log_2(x+1) + \log_2(x-2) = 2$   
 $\Rightarrow \log_2[(x+1)(x-2)] = 2 = \log_2 4 \Rightarrow (x+1)(x-2) = 4$   
 $\Rightarrow x^2 - x - 2 = 4 \Rightarrow (x-3)(x+2) = 0$

故  $x = 3$  或  $x = -2$  ( $\because$  真數  $> 0$ , 故不合)

5.  $\log_{10}(x^2 + x + 18) = \log_{10}(x+1) + \log_{10}10 = \log_{10}10(x+1)$   
 $\Rightarrow x^2 + x + 18 = 10x + 10 \Rightarrow x^2 - 9x + 8 = 0 \Rightarrow$   
 $(x-1)(x-8) = 0$   
 $\Rightarrow x = 1$  或  $x = 8$

6.  $\frac{1}{3} \log_{10} 216 = \frac{1}{3} \log_{10} 6^3 = \log_{10} 6$

$\frac{1}{4} \log_{10} 625 = \frac{1}{4} \log_{10} 5^4 = \log_{10} 5$

$\frac{1}{5} \log_{10} 243 = \frac{1}{5} \log_{10} 3^5 = \log_{10} 3$

所求

$$= \log_{10} 4 - \log_{10} 6 + \log_{10} 5 + \log_{10} 3 - \log_{10} \frac{4 \times 5 \times 3}{6} - \log_{10} 10$$

7. 所求  $= (\log_2 96 - \log_2 3) + (\log_3 405 - \log_3 5)$   
 $= \log_2 \frac{96}{3} + \log_3 \frac{405}{5}$   
 $= \log_2 32 + \log_3 81 = 5 + 4 = 9$

8.  $a = \log_5 2 \Rightarrow 5^a = 2$

$b = \log_3 5 \Rightarrow 3^b = 5 \Rightarrow (3^b)^{\frac{1}{b}} = 5^{\frac{1}{b}} \Rightarrow$

$3 = 5^{\frac{1}{b}}$

$\therefore 5^{a - \frac{1}{b} + 2} = 5^a \times 5^{-\frac{1}{b}} \times 5^2 = 2 \times \frac{1}{3} \times 25 = \frac{50}{3}$

9. 底數  $x-3 > 0$  且  $x-3 \neq 1 \Rightarrow x > 3$  且  $x \neq 4$

真數  $9-2x > 0 \Rightarrow 9 > 2x \Rightarrow \frac{9}{2} > x$

故  $3 < x < \frac{9}{2}$ , 且  $x \neq 4$

11. 所求 =  $(\log_3 5) \times (\log_5 2^5) \times (\log_2 81)$   
 =  $(\log_3 5) \times (5 \log_5 2) \times (\log_2 81)$   
 =  $5 \times (\log_3 5) \times (\log_5 2) \times (\log_2 81)$   
 =  $5 \times \log_3 81 = 5 \times 4 = 20$

12. (1) 令  $x > 0$  且  $x \neq 1$

原式 =  $\frac{\log_x 3}{\log_x 2} \times \frac{\log_x 4}{\log_x 3} \times \frac{\log_x 5}{\log_x 4} \times \frac{\log_x 6}{\log_x 5} \times \frac{\log_x 7}{\log_x 6}$

$\times \frac{\log_x 8}{\log_x 7}$   
 =  $\frac{\log_x 8}{\log_x 2} = \log_2 8 = \log_2 2^3 = 3$

(2) 原式 =  $\frac{\log_7 16}{\log_{\sqrt{49}} \sqrt{8}} = \frac{\log_7 16}{\log_7 \sqrt{8}} = \log_{\sqrt{8}} 16$

=  $\log_{2^{\frac{3}{2}}} 2^4 = \frac{4}{\frac{3}{2}} = \frac{8}{3}$

13. 所求 =  $\log_6 4 + \log_6 27 + \log_6 12$

=  $\log_6 (4 \times 27 \times 12) = \log_6 6^4 = 4$

14.  $\log_3 4 + \log_3 2 = 2 \log_3 2 + \log_3 2 = 3 \log_3 2$

$\log_2 3 + \log_4 27 = \log_2 3 + \frac{3}{2} \log_2 3 = \frac{5}{2} \log_2 3$

$\therefore$  原式 =  $3 \log_3 2 \times \frac{5}{2} \log_2 3 = 3 \times \frac{\log 2}{\log 3} \times \frac{5}{2} \times \frac{\log 3}{\log 2}$

=  $\frac{15}{2}$

15. (1) 原式 =  $\frac{\log_{7^2} (2^5)^2}{\log_{49} 2^3} = \frac{\log_{49} 2^{10}}{\log_{49} 2^3} = \log_{2^3} 2^{10} = \frac{10}{3}$

(2) 原式

=  $\log_3 \left(\frac{2}{9}\right)^3 - \log_3 \left(\frac{10}{3}\right)^2 + \log_3 \left(\frac{25}{6}\right) = \log_3 \frac{\frac{8}{729} \times \frac{25}{6}}{\frac{100}{9}} = \log_3 \frac{8 \times 25}{729 \times 6 \times 100} = 2 \log_3 5 + \log_3 4$

=  $\log_3 \frac{1}{243} = \log_3 3^{-5} = -5$

(3) 原式

=  $\log_5 5^{\frac{3}{2}} + \log_{2^3} 2^2 + \log_{3^3} 3 - \log_{10} 10^{-2} = \frac{3}{2} + \frac{2}{3} + \frac{1}{3} + 2 = \frac{9}{2}$

(4) 原式 =  $(\log_3 5^2)(\log_4 3)(\log_5 4^2) =$

$(2 \log_3 5)(\log_4 3)(2 \log_5 4)$

=  $4(\log_3 5)(\log_5 4)(\log_4 3) = 4 \log_3 3 = 4$

(5) 原式 =  $(\log_2 5 - \log_2 \frac{1}{5})(\log_5 2 + \log_5 \frac{1}{2})$

=  $(\log_2 5 - \frac{-1}{2} \log_2 5)(\log_5 2 + \frac{-1}{2} \log_5 2) = (\frac{3}{2} \log_2 5)(\frac{1}{2} \log_5 2)$

=  $\frac{3}{4} \log_2 5 \times \log_5 2 = \frac{3}{4}$

16.  $\log_{\frac{1}{16}} \frac{1}{2} + \log_{\frac{1}{25}} \frac{1}{5} + \log_{\frac{1}{27}} \frac{1}{3} = \frac{1}{4} + \frac{1}{2} + \frac{1}{3} = \frac{13}{12}$

17.  $2 \log_{10} \frac{5}{3} - \log_{10} \frac{7}{4} + 2 \log_{10} 3 + \frac{1}{2} \log_{10} 49$

=  $2(\log_{10} 5 - \log_{10} 3) - (\log_{10} 7 - \log_{10} 4) + 2 \log_{10} 3 + \frac{1}{2} \log_{10} 49$   
 =  $2 \log_{10} 5 - \log_{10} 7 + \log_{10} 4$   
 =  $2(\log_{10} 5 + \log_{10} 2) = 2$

18. 原式

=  $(\log_2 3 + \log_2 3)(2 \log_3 2 + \frac{1}{2} \log_3 2) = (2 \log_2 3)(\frac{5}{2} \log_3 2)$

=  $5 \times \log_2 3 \times \log_3 2 = 5$

20.  $\therefore$  底數  $4 > 1$  且  $\frac{1}{4} < \frac{1}{3} < \frac{1}{2}$

$\therefore \log_4 \frac{1}{4} < \log_4 \frac{1}{3} < \log_4 \frac{1}{2} \Rightarrow a < b < c$

21.  $\therefore$  底數  $0.4$  介於  $0$  與  $1$  之間且  $5 < 6 < 7$

$\therefore \log_{0.4} 5 > \log_{0.4} 6 > \log_{0.4} 7$

$\Rightarrow a > b > c \Rightarrow c < b < a$

22.  $\therefore$  底數  $5 > 1$

$$\therefore 3x < x+6 \Rightarrow x < 3 \cdots \cdots \textcircled{1}$$

而真數  $3x > 0$  和  $x+6 > 0$

$$\Rightarrow x > 0 \text{ 且 } x > -6 \Rightarrow x > 0 \cdots \cdots \textcircled{2}$$

由①與②得： $0 < x < 3$

$$23. \text{原式} \Rightarrow \log_{\frac{2}{3}}(2x-8) > \log_{\frac{2}{3}} \frac{2}{3} + \log_{\frac{2}{3}}(x+6)$$

$$\Rightarrow \log_{\frac{2}{3}}(2x-8) > \log_{\frac{2}{3}} \left[ \frac{2}{3}(x+6) \right]$$

$\therefore$  底數  $\frac{2}{3}$  介於 0 與 1 之間

$$\therefore 2x-8 < \frac{2}{3}(x+6) \Rightarrow x < 9 \cdots \cdots \textcircled{1}$$

而真數  $2x-8 > 0$  和  $x+6 > 0$

$$\Rightarrow x > 4 \text{ 且 } x > -6 \Rightarrow x > 4 \cdots \cdots \textcircled{2}$$

由①與②得： $4 < x < 9$

24.(1)  $\log_{0.3}(x-1)$  與  $\log_{0.3}(2x+4)$  有意義

$$\text{則 } x-1 > 0 \text{ 且 } 2x+4 > 0 \Rightarrow x > 1 \text{ 且 } x > -2$$

$$\Rightarrow x > 1$$

$$\log_{0.3}(x-1) > \log_{0.3}(2x+4) \Rightarrow x-1 < 2x+4$$

$$\Rightarrow -5 < x$$

故  $x > 1$

(2)  $\log_2 x$  有意義，則  $x > 0$

$$\frac{1}{2} < \log_2 x < 2 \Rightarrow \log_2 2^{\frac{1}{2}} < \log_2 x < \log_2 2^2$$

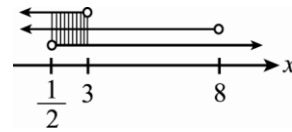
$$\Rightarrow 2^{\frac{1}{2}} < x < 2^2 \Rightarrow \sqrt{2} < x < 4$$

故  $\sqrt{2} < x < 4$

$$26. \log_{0.2}(3x-1) > 0 = \log_{0.2} 1 \Rightarrow 0 < 3x-1 < 1 \Rightarrow$$

$$\frac{1}{3} < x < \frac{2}{3}$$

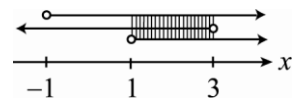
$$28. \begin{cases} 2x-1 > 0 \\ 8-x > 0 \\ 2x-1 < 8-x \end{cases} \Rightarrow \begin{cases} x > \frac{1}{2} \\ x < 8 \\ x < 3 \end{cases}$$



如圖所示  $\therefore \frac{1}{2} < x < 3$

29.(1)  $\therefore$  底數  $0 < \frac{1}{2} < 1$  為遞減函數

$$\therefore \begin{cases} 3-x < x+1 \Rightarrow x > 1 \\ 3-x > 0 \Rightarrow x < 3 \\ x+1 > 0 \Rightarrow x > -1 \end{cases}$$



如圖所示  $\therefore 1 < x < 3$

(2)  $\log_3(2x-3) < \log_3 9$

$\therefore$  底數  $3 > 1$  為遞增函數

$$\therefore \begin{cases} 2x-3 < 9 \Rightarrow x < 6 \\ 2x-3 > 0 \Rightarrow x > \frac{3}{2} \end{cases}$$



如圖所示  $\therefore \frac{3}{2} < x < 6$