

班級 _____ 班 座號 _____ 姓名 _____

答案

一、單選題 (16 題 每題 4 分 共 64 分)

1.B 2.D 3.A 4.C 5.D 6.B 7.D 8.D 9.D 10.A 11.D 12.A 13.A 14.C 15.B 16.D

二、填充題 (9 題 每題 4 分 共 36 分)

1.2 2. $\frac{1}{3}$, 1 3. $\frac{1}{2}$ 4. $x > 5$ 5. $b > a > c$ 6. $x < \frac{4}{3}$ 7. -1 或 2 8. $-\frac{13}{5}$ 9.4

解析

一、單選題 (16 題 每題 4 分 共 64 分)

2. 原式 $\Rightarrow 2(2^x)^2 + (2^x)^3 = 5 \times 2^4 \times 2^x$

 $\because 2^x$ 恆為正數

等號兩邊同除以 2^x , 得 $2 \times 2^x + (2^x)^2 = 80$

$\Rightarrow (2^x)^2 + 2 \times 2^x - 80 = 0 \Rightarrow (2^x + 10)(2^x - 8) = 0$

$\Rightarrow 2^x = 8$ 或 $2^x = -10$ (不合)

$\therefore x = 3$

3. $2^{2x+2} = 9 \times 2^x - 2 \Rightarrow 4 \times 2^{2x} - 9 \times 2^x + 2 = 0$

令 $2^x = k \Rightarrow 4k^2 - 9k + 2 = 0 \Rightarrow (4k-1)(k-2) = 0$

故 $k = \frac{1}{4}$ 或 2, 即 $x = -2$ 或 1

4. 令 $2^x = k \Rightarrow 2k^2 + k^3 = 80k \Rightarrow k(k^2 + 2k - 80) = 0$

$84 \cdot 3^{x-3} = 84 \cdot 3^x \cdot 3^{-3} = \frac{28}{9}t$

$\therefore \frac{1}{3}t^2 - \frac{28}{9}t + 1 = 0 \Rightarrow 3t^2 - 28t + 9 = 0$

$\Rightarrow t = \frac{1}{3}$ 或 9 $\Rightarrow 3^x = 3^{-1}$ 或 3^2

$\Rightarrow x = -1$ 或 2

15. $\frac{1}{9} < \left(\frac{1}{3}\right)^{2x+1} < 9 \Rightarrow 3^{-2} < (3^{-1})^{2x+1} < 3^2$

$\Rightarrow 3^{-2} < 3^{-2x-1} < 3^2 \Rightarrow -2 < -2x-1 < 2$

$\Rightarrow -1 < -2x < 3 \Rightarrow \frac{1}{2} > x > -\frac{3}{2}$

$\Rightarrow -\frac{3}{2} < x < \frac{1}{2}$

$\Rightarrow k(k-8)(k+10) = 0$

故 $k = -10, 0, 8$ (但 $-10, 0$ 不合), 即 $x = 3$

5. 原式 $\Rightarrow (3^x)^2 - 4 \times 3^x - 45 = 0 \Rightarrow (3^x - 9)(3^x + 5) = 0$

 $\therefore 3^x = 9$ 或 $3^x = -5$ (不合), 故 $x = 2$

6. $(0.3)^x > (0.3)^4$ 故 $x < 4$

7. 原式 $\Rightarrow 2^{2(6-x)} > 2^{x+2} \Rightarrow 12 - 2x > x + 2 \Rightarrow 3x < 10 \Rightarrow x < \frac{10}{3}$

8. 原式 $\Rightarrow (0.3)^{3x+1} > (0.3)^{10-6x} \Rightarrow 3x+1 < 10-6x \Rightarrow 9x < 9$
 $\Rightarrow x < 1$

9. $\because 0.008 = (0.2)^3 \therefore (0.2)^x > (0.2)^3$

又底數為 $0.2 < 1$, 故為遞減函數

$\Rightarrow x < 3$

10. \because 底數 $0.7 < \frac{7}{9} < 0.9 \Rightarrow (0.9)^{15} > \left(\frac{7}{9}\right)^{15} > (0.7)^{15} \Rightarrow b > c > a$

11. \because 底數 $1.3 < \frac{4}{3} < 1.4 \Rightarrow (1.3)^{10} < \left(\frac{4}{3}\right)^{10} < (1.4)^{10} \Rightarrow b < c < a$

13. $a = \sqrt{2} = 2^{\frac{1}{2}}$, $b = \sqrt[4]{8} = 8^{\frac{1}{4}} = 2^{\frac{3}{4}}$, $c = \frac{1}{4} = 4^{-1} = 2^{-2}$

底數為 $2 > 1$, 故為遞增函數

又指數 $\frac{3}{4} > \frac{1}{2} > -2 \Rightarrow 2^{\frac{3}{4}} > 2^{\frac{1}{2}} > 2^{-2}$

故 $b > a > c$

14. 令 $t = 3^x > 0$, $3^{2x-1} = \frac{1}{3} \times (3^x)^2 = \frac{1}{3}t^2$

$$16. (0.2)^{x^2-2x-5} < 0.008 = (0.2)^3$$

$$\Rightarrow x^2 - 2x - 5 > 3 \Rightarrow (x-4)(x+2) > 0$$

$$\Rightarrow x < -2 \text{ 或 } x > 4$$

二、填充題 (9 題 每題 4 分 共 36 分)

$$1. 4 \times 2^{2x} - 3 \times 4 \times 2^x - 16 = 0 \Rightarrow (2^x)^2 - 3 \times 2^x - 4 = 0$$

$$\because 2^x = 4 \text{ 或 } 2^x = -1 \text{ (不合)} \quad \therefore x = 2$$

$$3. 2^{2m} = \frac{1}{64} = 2^{-6} \Rightarrow m = -3, 3^{-3-2n} = 81 = 3^4 \Rightarrow n = -\frac{7}{2}$$

$$\text{故 } m - n = (-3) - \left(-\frac{7}{2}\right) = \frac{1}{2}$$

$$4. (0.2)^{x+1} > (0.2)^{2x-4} \Rightarrow x+1 < 2x-4 \Rightarrow x > 5$$

$$5. a = \sqrt[4]{27} = 3^{\frac{3}{4}}, b = \sqrt[5]{81} = 3^{\frac{4}{5}}, c = \sqrt[3]{9} = 3^{\frac{2}{3}}$$

$$\because \frac{4}{5} > \frac{3}{4} > \frac{2}{3} \quad \therefore b > a > c$$

$$6. (0.008)^x > 0.0016 \Rightarrow (0.2)^{3x} > (0.2)^4$$

$$\text{故 } 3x < 4 \quad \therefore x < \frac{4}{3}$$

$$7. 4^x - 9 \times 2^{x-1} + 2 = 0$$

$$\Rightarrow (2^x)^2 - \frac{9}{2} \times 2^x + 2 = 0 \Rightarrow 2 \times (2^x)^2 - 9 \times 2^x + 4 = 0$$

$$\Rightarrow (2 \times 2^x - 1)(2^x - 4) = 0 \Rightarrow 2^x = \frac{1}{2} \text{ 或 } 2^x = 4 \Rightarrow x = -1 \text{ 或 } x = 2$$

$$8. \left(\frac{1}{2}\right)^{3-x} = 4^{3x+5}$$

$$\Rightarrow (2^{-1})^{3-x} = (2^2)^{3x+5} \Rightarrow x-3 = 2(3x+5) \Rightarrow -13 = 5x$$

$$\text{故 } x = -\frac{13}{5}$$

$$9. \begin{cases} (3^x)^2 = 81 \\ 2^{x-3y} = 256 \end{cases} \Rightarrow \begin{cases} 3^{2x} = 3^4 \\ 2^{x-3y} = 2^8 \end{cases} \Rightarrow \begin{cases} 2x = 4 \\ x - 3y = 8 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = -2 \end{cases}$$

$$\text{故 } x - y = 2 - (-2) = 4$$